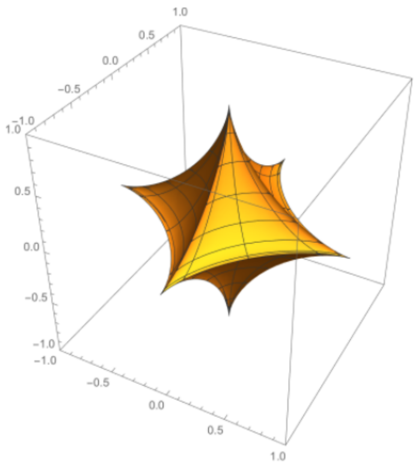
GAMO
DAY 1

Gaussian Universal Standard Actual Mathematical Olympiad

- ☐ Each problem is worth 7 points. There is negative marking and there is partial marking.
- ☐ Any type of fake solve or proof is highly discouraged, it will result in loss of your marks.
- ☐ Note that the use of Barycentric co-ordinates, Complex Numbers, Moving points or Co-ordinate geometry in solving geometry problems does not result in a loss of points. Note that 1 point will be deducted if the diagram for a geometry problem if and when required is not drawn.
- ☐ Submission Deadline is 11th April 2021. Submit your subjective solutions to Aritra12, TLP.39, Orestis_Lignos, EpicNumberTheory, Phoenixfire,i3435 added in one PM on AoPS PM.
- ☐ The Search Function won't help you since all problems are original. If you do find any problem that is not original PM it to us immediately.



GAUSSIAN 1st USAMO 2021

Day 1

2nd April, 2021

A-1. Let $a, b > 1$ be any two distinct positive integers. You are given these two quantities:

$$\frac{a}{1} \text{ and } \frac{1}{a}$$

You are allowed to apply any of these three operations:

- You can add or subtract any positive integer onto/from the numerators of both the quantities (simultaneously).
- You can add or subtract any positive integer onto/from the denominators of both the quantities (simultaneously)
- You can reduce any quantity to it's lowest form (You need not reduce both quantities simultaneously).
- You can interchange the position of the two quantities.

Note that the numerator is always non-negative and the denominator positive at any point. Determine whether it is possible to attain the following configuration of $\frac{b}{1}$ and $\frac{1}{b}$ from the current one by a finite (possibly empty) sequence of such operations.

A-2. Find all polynomials $P(x)$ with integer coefficients which satisfy the following conditions

- $P(n)$ is a positive integer for any positive integer n.
- $P(n)!$ divides $\prod_{k=1}^{n}\left(2^{P(k)+k-1} - 2^{k-1}\right)$ for all positive integers n.

A-3. Let ABC be a triangle with incenter I and Nagel Point N. Let N' be the reflection of N on BC. Let D be on the circumcircle of ABC such that $AD \perp BC$. Let the circle with diameter AI intersect the circumcircle of ABC at $S \neq A$. Let M be the midpoint of the arc BC not containing A and let AN intersect the circumcircle of ABC at X. Then MX, BC and the perpendicular from N' onto SD concur.

Note: The Nagel point of a triangle ABC is defined as the intersection point of the cevians joining the corresponding vertex to the point where the respective excircle touch the side opposite to that vertex

Language: English

Each problem is worth 7 points

4 hours and 30 minutes only