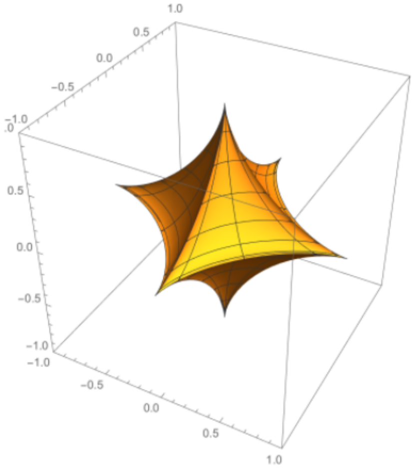
GAMO

DAY 2

Gaussian Universal Standard Actual Mathematical Olympiad

- ☐ Each problem is worth 7 points. There is negative marking and there is partial marking.
- ☐ Any type of fake solve or proof is highly discouraged, it will result in loss of your marks.
- ☐ Note that the use of Barycentric co-ordinates, Complex Numbers, Moving points or Co-ordinate geometry in solving geometry problems does not result in a loss of points. Note that 1 point will be deducted if the diagram for a geometry problem if and when required is not drawn.
- ☐ Submission Deadline is 11th April 2021. Submit your subjective solutions to Aritra12, TLP.39, Orestis_Lignos, EpicNumberTheory, Phoenixfire,i3435 added in one PM on AoPS PM.
- ☐ The Search Function won't help you since all problems are original. If you do find any problem that is not original PM it to us immediately.



GAUSSIAN 1st USAMO 2021

Day 2

3rd April, 2021

A-4. Let $n \geq 1$ be a positive integer, and let $\mathcal{S} \subset 0, 1, 2, \ldots, n$ such that

$$|\mathcal{S}| \geq \frac{n}{2} + 1.$$

Show that some power of 2 is either an element of $\mathcal{S}$ or the sum of two distinct elements of $\mathcal{S}$.

A-5. Let ABC be an acute, non-isosceles triangle, AD, BE, CF be its heights and (c) its circumcircle. FE cuts the circumcircle at points S, T, with point F being between points S, E. In addition, let P, Q be the midpoints of the major and the minor arc BC, respectively. Line DQ cuts (c) at R. The circumcircles of triangles RSF, TER, SFP and TEP cut again PR at points X, Y, Z and W, respectively. Suppose (ℓ) is the line passing through the circumcenters of triangles AXW, AYZ and $(\ell_B), (\ell_C)$ the parallel lines through B, C to (ℓ). If (ℓ_B) meets CF at U and (ℓ_C) meets BE at V, then prove that points U, V, F, E are concyclic.

A-6. Find all functions $f : \mathbb{Z} \to \mathbb{Z}$ such that for all integers x, y,

$$f(x^2 + f(y)) + f(yf(x)) = f(x)f(x + y) + f(y)$$

Language: English

Each problem is worth 7 points

4 hours and 30 minutes only