# GJMO
DAY 1

Gaussian Universal Standard Actual Junior Mathematical Olympiad

☐ Each problem is worth 7 points. There is negative marking and there is partial marking.

☐ Any type of fake solve or proof is highly discouraged, it will result in loss of your marks.

☐ Note that the use of Barycentric co-ordinates, Complex Numbers, Moving points or Co-ordinate geometry in solving geometry problems does not result in a loss of points. Note that 1 point will be deducted if the diagram for a geometry problem if and when required is not drawn.

☐ Submission Deadline is 11th April 2021. Submit your subjective solutions to Aritra12, TLP.39, Orestis_Lignos, EpicNumberTheory, Phoenixfire,i3435 added in one PM on AoPS PM.

☐ The Search Function won't help you since all problems are original. If you do find any problem that is not original PM it to us immediately.
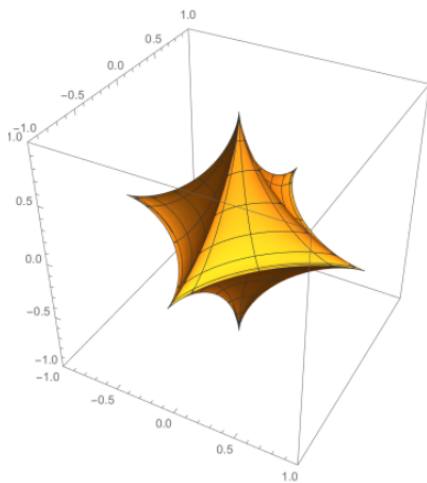
J-1. Find the minimum possible value of the natural number x, such that:

- $x > 2021$ and
- There is a positive integer y, co-prime with x, such that $x^2 - 4xy + 5y^2$ is a perfect square

J-2. In phoenix, a Galaxy far, far away, there are 2021 planets. Define a fire to be a path between two objects in phoenix. It is known that between every pair of planets either a single fire burns or no burning occurs. If we consider any subset of 2019 planets, the total number of fires burning between these planets is a constant. If there are $\mathcal{F}$ fires in phoenix, then find all possible values of $\mathcal{F}$.

J-3. Let ABC be a triangle with sides a, b, c and let r_a, r_b, r_c denote the radii of the excircles of triangle ABC. If R denotes the circumradius of triangle ABC then prove that

$$\frac{4[ABC]}{\sqrt{ab} + \sqrt{bc} + \sqrt{ca}} \le R^2 \sum \frac{h_c}{r_a \cdot r_b} \left(\cos \frac{A}{2} \right)^4$$

where h denotes altitude , $[x]$ denotes area of x