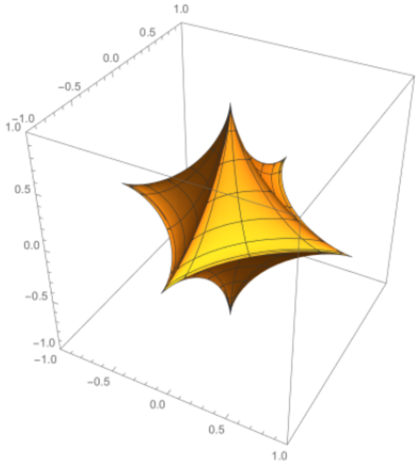
GJMO

DAY 2

Gaussian Universal Standard Actual Junior Mathematical Olympiad

- ☐ Each problem is worth 7 points. There is negative marking and there is partial marking.
- ☐ Any type of fake solve or proof is highly discouraged, it will result in loss of your marks.
- ☐ Note that the use of Barycentric co-ordinates, Complex Numbers, Moving points or Co-ordinate geometry in solving geometry problems does not result in a loss of points. Note that 1 point will be deducted if the diagram for a geometry problem if and when required is not drawn.
- ☐ Submission Deadline is 11th April 2021. Submit your subjective solutions to Aritra12, TLP.39, Orestis_Lignos, EpicNumberTheory, Phoenixfire,i3435 added in one PM on AoPS PM.
- ☐ The Search Function won't help you since all problems are original. If you do find any problem that is not original PM it to us immediately.



GAUSSIAN 1st USAJMO 2021

Day 2

3rd April, 2021

J-4. On the board n positive integers are written, let them be $a_1, a_2, \ldots, a_n$. Let p, q be two prime numbers such that $p \neq q$. We are allowed to execute infinitely many times the following procedure: We pick two numbers a, b written on the board, we delete them and replace them with $pa - qb, pb - qa$. After 2021 applications of this procedure, let k be the product of all numbers on the board that time. If we know that $k^{(p-1)(q-1)} \not\equiv 1 \pmod{pq}$, then prove that there exists a $i \in \{1, 2, \ldots, n\}$, such that either $p|a_i$ or $q|a_i$.

J-5. In a $\triangle ABC$, let K be the intersection of the A-angle bisector and $\overline{BC}$. Let H be the orthocenter of $\triangle ABC$. If the line through K perpendicular to $\overline{AK}$ meets $\overline{AH}$ at P, and the line through H parallel to $\overline{AK}$ meets the A-tangent of (ABC) at Q, then prove that $\overline{PQ}$ is parallel to the A-symmedian.

Note: The A-symmedian is the reflection of the A-median over the A-angle bisector.

J-6. Let $S = \{1, 2, \ldots, n\}$, with $n \geq 3$ being a positive integer. Call a subset A of S *gaussian* if $|A| \geq 3$ and for all $a, b, c \in A$ with $a > b > c$,

$$\frac{a^2}{b^2} + \frac{b^2}{c^2} + \frac{c^2}{a^2} < 5$$

holds true.

(i) Prove that $|A| \leq \left\lfloor \frac{n+2}{2} \right\rfloor$ for all gaussian subsets A of S.

(ii) If a gaussian subset of S contains exactly $\left\lfloor \frac{n+2}{2} \right\rfloor$ elements, then find all possible values of n.

Language: English

Each problem is worth 7 points

4 hours and 30 minutes only